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Gauge theoretic approach to the electromagnetic response of the high-temperature superconductors

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Abstract. The gauge theoretic approach to the t–J model provides a tool to investigate its electromagnetic properties and hence those of the cuprate superconductors, which are modelled by this Hamiltonian. In a previous letter we presented a calculation, within the gauge field approach, of the Hall effect. In this paper we shall explain in greater detail the method used and extend this analysis to provide the framework from which higher-order effects, such as the magnetoresistance, can be calculated. We also consider a further set of terms (the 'Gaussian fluctuations'), expanding upon those published.

1. Introduction

The electromagnetic properties of the normal state of the high-T_c superconductors (HTSC) display, for example, unusual temperature dependencies when compared with those of a normal Fermi liquid, e.g. the temperature variations of the in-plane conductivity [1, 2] and Hall angle [3, 4]. The physics of the HTSC in such situations is believed to be essentially that of the electrons in the copper–oxygen planes, for which the proposed microscopic Hamiltonian is the two-dimensional, repulsive and single-band Hubbard model on a square lattice [5, 6], in the strong correlation limit ($U \gg t$). This Hamiltonian may be further approximated by the t–J model [7, 8], an effective Hamiltonian in the constrained subspace of no double occupancy, with the constraint dealt with by use of the slave particle representation [9, 10, 11]. The low energy, continuum and imaginary time Lagrangian density reads [12, 13]:

$$\mathcal{L} = \sum_{a=0,\uparrow,\downarrow} \bar{y}^a(x)\left(\partial_\tau - \mu_a + \frac{1}{2m_a}[-\mathrm{i}\nabla + \boldsymbol{a}(x)]^2\right) y^a(x) \tag{1}$$

where $\bar{y}^0, y^0$ is the bosonic 'holon' field, $\bar{y}^\alpha, y^\alpha$ ($\alpha = \uparrow, \downarrow$) are the fermionic 'spinon' fields and $\boldsymbol{a}$ is a $U(1)$ 'internal' gauge reflecting the local gauge symmetry present in the original t–J Hamiltonian in the slave particle representation, where the temporal gauge, i.e. $a_0 = 0$, has been chosen.

The electromagnetic response of the system described by (1) is, in principle, straightforward to analyse and may be approached by two methods, in both of which only one slave, which we shall take to be the holon, carries physical charge. One of these methods is that of Ioffe and Kotliar [14] and Lee and Nagaosa [13, 15, 16], based on the result that the sum of the holon and spinon currents, $\boldsymbol{J}^B$ and $\boldsymbol{J}^F$, respectively, is zero (the 'current constraint'), with these currents also related to the physical electron current ($\boldsymbol{J}$)

by $J = J^F = -J^B$. Taking the slaves to possess unphysical analogues of the physical responses, the slave currents are then driven by either the internal electric and magnetic fields, from a, if uncharged, or by these plus the external fields, if charged. Thus, given the slave currents, the electron current follows from the above current relationships and hence its responses under the external fields. In the other approach we again determine the electronic current, but now from the effective action of the external vector potential (A), $S_{\rm eff}[A]$, taken to be in the same gauge as the internal field; this is the method pioneered by Ioffe and Larkin [17]. With the holons charged, thereby interacting with the field $a + \tilde{A}$ (where $\tilde{A} = -eA/c$, e being the magnitude of the electronic charge and c the speed of light [18]), it follows that the charge current is given by $-c\,\delta S_{\rm eff}/\delta A$ [18], from which the response can be derived. In this article we shall be concerned with this second method, though we will also consider the former approach for the sake of comparison.

In principle we may derive $S_{\rm eff}$ by first integrating out the slave fields, as $\mathcal{L}$ is quadratic in these, yielding an action for just a and A, which we shall denote by $S[a, A]$, followed by integrating over a. However, S is not quadratic in a, but also contains higher powers of a, so this functional integration cannot be performed exactly. Instead, approximations must be used and we shall consider the first two terms in what is essentially a power series in $\hbar$: (a) the saddle-point action, which is $O(1/\hbar)$, described in section 2 and (b) the Gaussian fluctuation contribution, $O(\hbar^0)$ [19], described in section 3 and discussed in section 4. We shall find that the electronic susceptibilities can be expressed in terms of the slave susceptibilities. In [20], an 'Ioffe–Larkin' combination formula was derived through a first-principles calculation at the saddle-point level alone. A similar calculation was performed in [21], with the Gaussian fluctuation contribution included, but not completely, as we shall explain below.

It is important to note that in calculating $S_{\rm eff}$ we will do so in powers of two quantities: $\hbar$, as discussed above (although explicitly we take $\hbar = 1$) and e/c or, equivalently, powers of the vector potential. Furthermore, our approach will be to calculate, at each order in $\hbar$, correct to some power (order) of A and we shall use order in this second sense when explaining the calculation of the terms in the $\hbar$ expansion.

2. The saddle-point action

The saddle point, or classical action ($S^c_{\rm eff}[A]$) is defined by

$$S^c_{\rm eff}[A] = S[a^c, A] \qquad \text{where} \quad \left.\frac{\delta S[a^c, A]}{\delta a(x)}\right|_{a=a^c} = 0 \tag{2}$$

where $a^c = a^c[A]$ is the classical solution for the internal gauge-field in the presence of the external gauge-field. In any calculation of $S_{\rm eff}$ we must work to some power of A. To be concrete, let us take $S[a, A]$ through to fourth-order terms as this introduces no extra complications and illustrates the method. Therefore:

$$\begin{aligned} S[a, A] = {} & \tfrac{1}{2}\left[\Pi^F_{ab} a_a a_b + \Pi^B_{ab}(a_a + \tilde{A}_a)(a_b + \tilde{A}_b)\right] \\ & + \tfrac{1}{3}\left[\Gamma^F_{abc} a_a a_b a_c + \Gamma^B_{abc}(a_a + \tilde{A}_a)(a_b + \tilde{A}_b)(a_c + \tilde{A}_c)\right] \\ & + \tfrac{1}{4}\left[\Delta^F_{abcd} a_a a_b a_c a_d + \Delta^B_{abcd}(a_a + \tilde{A}_a)(a_b + \tilde{A}_b)(a_c + \tilde{A}_c)(a_d + \tilde{A}_d)\right] \end{aligned} \tag{3}$$

where each subscript is a coordinate and a discrete element, so $A_a \leftrightarrow A_\alpha(x)$, with repeated subscripts indicating an integral over the coordinate and a sum over the discrete element.

The Πs have been described in [12, 13] and the Γs in [20, 21, 22], but the four-field vertices have not been previously discussed and are given by

$$\Delta^a_{abcd} = (1/3!)\Big[-\langle J^a_a J^a_b J^a_c J^a_d\rangle_c + (1/m_a)\big(\langle J^a_a J^a_b \rho_{cd}\rangle_c + \langle J^a_a J^a_c \rho_{bd}\rangle_c + \langle J^a_a J^a_d \rho_{bc}\rangle_c + \langle J^a_b J^a_c \rho_{ad}\rangle_c + \langle J^a_b J^a_d \rho_{ac}\rangle_c + \langle J^a_c J^a_d \rho_{ab}\rangle_c\big) - (1/m_a^2)\left(\langle \rho_{ab}\rho_{cd}\rangle_c + \langle \rho_{ac}\rho_{bd}\rangle_c + \langle \rho_{ad}\rho_{bc}\rangle_c\right)\Big] \tag{4}$$

where the superscript $a = 0, \uparrow, \downarrow$, as in (1) while $\langle\ldots\rangle_c$ indicates that only connected graphs are present and $\rho_{ab} \leftrightarrow \rho(x_1)\delta_{\lambda\mu}\delta(x_1 - x_2)$, ρ^a and J^a being the conventional number and current densities for the a-slave. Note also that $\Delta^B = \Delta^0$, while $\Delta^F = \Delta^\uparrow + \Delta^\downarrow$, which also holds for the other vertices.

Following Schwinger [23] we express $\boldsymbol{a}^c$ as a power series in $\boldsymbol{A}$ and solve the saddle-point equation in (2) by equating each power of $\boldsymbol{A}$ to zero, assuming $\boldsymbol{A}$ to be arbitrary. To be more specific, we take

$$a^c_a = \gamma^{(1)}_{ab} A_b + \gamma^{(2)}_{abc} A_b A_c + O(A^3) \tag{5}$$

and find that $\delta S/\delta \boldsymbol{a} = \boldsymbol{0}$ is solved by

$$\gamma^{(1)}_{ab} = -[\mathcal{D}\Pi^B]_{ab} \tag{6}$$

$$\gamma^{(2)}_{abc} = -\mathcal{D}_{ad}\left[\Gamma^B_{def}[\mathcal{D}\Pi^F]_{eb}[\mathcal{D}\Pi^F]_{fc} + \Gamma^F_{def}[\mathcal{D}\Pi^B]_{eb}[\mathcal{D}\Pi^B]_{fc}\right] \tag{7}$$

where $\mathcal{D} = \Pi^{-1}$, with $\Pi = \Pi^B + \Pi^F$, i.e. it is the internal gauge field propagator. It follows that the classical action may then be written [24, 25]

$$S^c_{\text{eff}}[A] = \tfrac{1}{2}\left[e/c\right]^2 \Pi^c_{ab} A_a A_b - \tfrac{1}{3}\left[e/c\right]^3 \Gamma^c_{abc} A_a A_b A_c + \tfrac{1}{4}\left[e/c\right]^4 \Delta^c_{abcd} A_a A_b A_c A_d \tag{8}$$

to fourth order in $\boldsymbol{A}$, where the vertices of the classical action, Π^c, Γ^c and Δ^c, are given by:

$$\Pi^c_{ab} = \Pi^{(2)}_{ab} \tag{9}$$

$$\Gamma^c_{abc} = \Gamma^{(3)}_{abc} \tag{10}$$

$$\Delta^c_{abcd} = \Delta^{(4)}_{abcd} - 2\Gamma^{(2)}_{iab}\mathcal{D}_{ij}\Gamma^{(2)}_{jcd} \tag{11}$$

defining:

$$\Pi^{(2)} = \sum_{s=B,F} \zeta_s^2 \Pi^s_{ef}[\mathcal{D}\Pi^{\bar{s}}]_{ea}[\mathcal{D}\Pi^{\bar{s}}]_{fb} \tag{12}$$

$$\Gamma^{(1)}_{abc} = \sum_{s=B,F} \zeta_s \Gamma^s_{abf}[\mathcal{D}\Pi^{\bar{s}}]_{fc} \tag{13}$$

$$\Gamma^{(2)}_{abc} = \sum_{s=B,F} \zeta_s^2 \Gamma^s_{aef}[\mathcal{D}\Pi^{\bar{s}}]_{fc}[\mathcal{D}\Pi^{\bar{s}}]_{eb} \tag{14}$$

$$\Gamma^{(3)}_{abc} = \sum_{s=B,F} \zeta_s^3 \Gamma^s_{def}[\mathcal{D}\Pi^{\bar{s}}]_{fc}[\mathcal{D}\Pi^{\bar{s}}]_{eb}[\mathcal{D}\Pi^{\bar{s}}]_{da} \tag{15}$$

and $\Delta^{(n)}$ $(n = 1, \ldots, 4)$ similarly, where $\bar{s} = B(F)$ and $\zeta_s = -1(+1)$ if $s = F(B)$; Δ^c is shown diagrammatically in figure 1. Note that the classical vertices have the form of a sum of slave vertices, with which the external field interacts via screening terms, i.e. the factor $\mathcal{D}\Pi^s$, established by the slave field of the other species; these screening factors also appear in the Gaussian corrections.

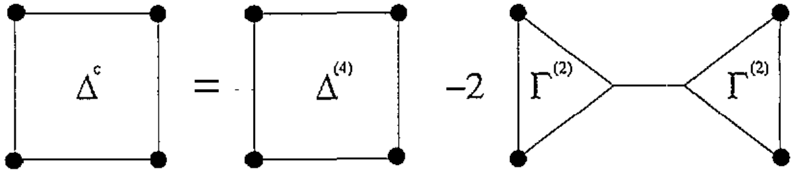

Figure 1. The fourth-order vertex in the saddle-point action. The dotted line is the internal gauge-field propagator and the vertices are labelled as in the text; the solid disc indicates the presence of a screening factor.

Truncating S at the nth-order terms, $\boldsymbol{a}^c$ can be solved for up to terms in the $(n-1)$th power of $\boldsymbol{A}$, to obtain S^c_{eff} to nth order. If we next consider S to $(n+1)$th order, then $\delta S/\delta a$ is as before plus an nth-order term, which cannot affect the lower-order terms of $\boldsymbol{a}^c$ and so the S^c_{eff} correct to nth and $(n+1)$th order only differ by an extra term in the latter, of order $(n+1)$th in $\boldsymbol{A}$. Thus, given S correct to nth order in $\boldsymbol{a}$ and $\boldsymbol{A}$, S^c_{eff} is also correct, in $\boldsymbol{A}$, to nth order, which is not the case when the Gaussian fluctuations are included (see below).

The second-order term in S^c_{eff} is that found by Ioffe and Larkin [17], from which the diamagnetic susceptibility (χ_{eff}) and the conductivity (σ_{eff}) of the electrons can be calculated [15, 26]. It follows that the conductivity, when the holon conductivity is much less than that of the spinon's, varies with temperature as $\sigma_{\mathrm{eff}} \sim T^{-1}$, which agrees with the experimental results that are typical for optimally doped samples [2]. The third-order term has been given in [20, 21] in relation to the Hall effect, but as noted therein, the simplest calculation for $\cot\theta_H$, the cotangent of the Hall angle (θ_H) was unable to reproduce the observed temperature and doping (δ) dependency of the superconducting cuprates, $\cot\theta_H = aT^2 + b(\delta)$, where $b(\delta) \simeq b\delta$ [3, 4]. The fourth-order term will be related to the (transverse) magnetoresistance, as this effect is $\propto EB^2$, along with non-linear corrections to the conductivity, presumably present in the third-order term too.

To derive the magnetoresistivity from the action we must first calculate the magnetoconductivity (σ^{eff}_M), where $\sigma_{\mathrm{eff}}(B) = \sigma^{\mathrm{eff}}_0 + \sigma^{\mathrm{eff}}_M B^2$, σ^{eff}_0 being the magnetic field independent part of the conductivity, previously denoted by σ_{eff}. This may be achieved by use of the same device employed in the study of the Hall effect [20, 27] of taking $\boldsymbol{A} = \boldsymbol{A}^{(1)} + \boldsymbol{A}^{(2)}$, where $\mathrm{i}\omega \boldsymbol{A}^{(1)}/c = (E, 0, 0)$ and $\mathrm{i}\boldsymbol{k} \times \boldsymbol{A}^{(2)} = (0, 0, B)$, and examining just that part of the current proportional to $A^{(2)}A^{(2)}A^{(1)}$. We have not attempted to calculate the Δ^s, but we assume that $\Delta^s \propto -\sigma^s_M/3$ [28]. Now, it can be shown, by calculating the Γ^s as before, that $\Gamma^s_{\alpha\beta\gamma}(\boldsymbol{k}, 0; \boldsymbol{k}, 0; -2\boldsymbol{k}, 0) = 0$, which corresponds to the observation that there is no current term in B^2 alone and so the contribution to the current form the second term in Δ^c is $-4\Gamma^{(2)}_{iab}\mathcal{D}_{ij}\Gamma^{(2)}_{jcd}A^{(2)}_b A^{(2)}_c A^{(1)}_d$. Therefore, if $\Pi^c_{\alpha\beta}(\omega, 0) = -\mathrm{i}\omega\sigma^{\mathrm{eff}}_0\delta_{\alpha\beta}$, $\omega \simeq 0$, then

$$\sigma^{\mathrm{eff}}_M = \frac{e^4}{(c\sigma_0\chi)^2}\left[(\sigma^F_0\chi_F)^2\sigma^B_M + (\sigma^B_0\chi_B)^2\sigma^F_M + \frac{1}{\sigma_0}\left(\sigma^F_0\chi_F\sigma^B_H + \sigma^B_0\chi_B\sigma^F_H\right)^2\right] \tag{16}$$

σ^s_0 being the magnetic-field-independent contribution to the s-slave conductivity and $\sigma_0 = \sigma_B + \sigma_F$, while $\chi = \chi_B + \chi_F$. The magnetoresistivity can now be calculated by inversion of the conductivity tensor; if $\sigma_{11} = \sigma_{22} = \sigma$, where $\sigma = \sigma_0 + \sigma_M B^2$ and $\sigma_{21} = -\sigma_{12} = \sigma_H B$, then for weak magnetic fields, the diagonal components of the resistivity are $\rho = \rho_0 + \Delta\rho B^2$, with $\rho_0 = 1/\sigma_0$ and $\Delta\rho = -(\sigma_M/\sigma_0 + (\sigma_H/\sigma_0)^2)/\sigma_0$, the latter defining the magnetoresistivity. Substituting into this expression the effective conductivities, we find that for $\sigma_M^{\rm eff}$ given by (16) we obtain

$$\Delta\rho_{\rm eff} = \frac{1}{c^2}\frac{\chi_F^2\Delta\rho_B + \chi_B^2\Delta\rho_F}{(\chi_B+\chi_F)^2} \tag{17}$$

where we have defined $\Delta\rho_s = -(\sigma_M^s/\sigma_0^s + (\sigma_H^s/\sigma_0^s)^2)/\sigma_0^s$, $s = B, F$. Alternatively, the current constraint argument could have been used and has been so by Nagaosa and Lee [16] and Ioffe and Wiegmann [29] who found (17) previously. Thus, as with the Hall effect, we observe that the effective action, at the classical level, leads to the same result as that obtained by the current constraint approach. Henceforth, we again again denote $\sigma_0^{\rm eff}$ and σ_0^s, respectively, by $\sigma_{\rm eff}$ and σ_s.

3. Derivation of the Gaussian correction

To include the Gaussian fluctuations the term

$$S_{\rm eff}^g := \frac{1}{2}{\rm Tr}\left\{\ln\left[\left.\frac{\delta^2 S}{\delta a^2}\right|_{a=a^c}\right]\right\} \tag{18}$$

is added to $S_{\rm eff}^c$, which, as explained in the introduction, should be smaller than $S_{\rm eff}^c$ by a factor $O(\hbar)$, but this will not be apparent, as we take $\hbar = 1$. From (18) it is clear that if $S_{\rm eff}$ is required to nth order in A, it is necessary to employ S correct to $(n+2)$th order so that $\delta^2S/\delta a^2$ is nth order, as desired. Consequently, to study the Hall effect, for which $S_{\rm eff}$ to third order is needed [20, 21], $S_{\rm eff}^g$ must be determined from S with terms up to fifth order in the gauge fields explicitly included, if we are to include all the relevant terms consistently, with respect to powers of A (or e/c).

Taking S to fifth order we find that $\delta^2S/\delta a^2$ is of the form $\Pi(1+2\mathcal{D}[\ldots])$, which we expand using the Taylor series for $\ln(1+x)$, keeping all terms to third order in A. Doing so, we obtain the result

$$S_{\rm eff}^g[A] = \tfrac{1}{2}{\rm Tr}\,\{\ln\Pi\} + \tfrac{1}{2}\Pi_{ij}^g\tilde{A}_i\tilde{A}_j + \tfrac{1}{3}\Gamma_{ijk}^g\tilde{A}_i\tilde{A}_j\tilde{A}_k \tag{19}$$

introducing the Gaussian corrections Π^g and Γ^g to the two- and three-field vertices, respectively [30].

Explicitly, the two-field contribution is

$$\Pi_{ij}^g = 3\mathcal{D}_{ab}\Delta_{baij}^{(2)} - 2\Gamma_{bci}^{(1)}\mathcal{D}_{ba}\mathcal{D}_{cd}\Gamma_{daj}^{(1)} \tag{20}$$

shown diagrammatically in figure 2, which has the form of Δ^c with the 'a, b' screening factors removed and replaced by a gauge propagator connecting these two points instead. From (20) we see that by including all the terms that are second order in A we obtain not only the corrections in [21], but also the diagrams which have so far been considered for the holons alone in [31], which may be seen as the lowest-order perturbation corrections to Π arising from the interaction terms in the starting Lagrangian.

Letting E_{abcde} denote the five point vertex, the three-field Gaussian correction is,

$$\begin{aligned}\Gamma_{ijk}^g = {} & 6E_{kbaij}^{(3)}\mathcal{D}_{ab} - 3\left[3\mathcal{D}_{ab}\Delta_{baei}^{(1)} - 2\Gamma_{bci}^{(1)}\mathcal{D}_{ba}\mathcal{D}_{cd}\Gamma_{dae}\right]\mathcal{D}_{ef}\Gamma_{fjk}^{(2)} \\ & -9\Gamma_{bci}^{(1)}\mathcal{D}_{ba}\mathcal{D}_{cd}\Delta_{dajk}^{(2)} + 4\mathcal{D}_{ab}\Gamma_{bci}^{(1)}\mathcal{D}_{cd}\Gamma_{dej}^{(1)}\mathcal{D}_{ef}\Gamma_{fak}^{(1)}\end{aligned} \tag{21}$$

where the terms containing Δ or E are new, not being included in [21] and are displayed in figure 3. Note that while Γ_{ijk}^g is not explicitly symmetric, it may readily be constructed to be so, but we do not give the result here.

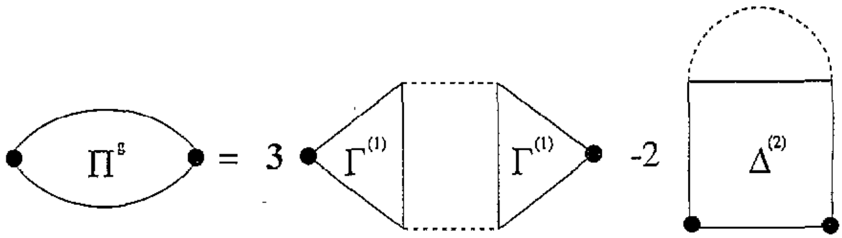

Figure 2. The Gaussian correction to the second-order vertex, with the same notation as for figure 1.

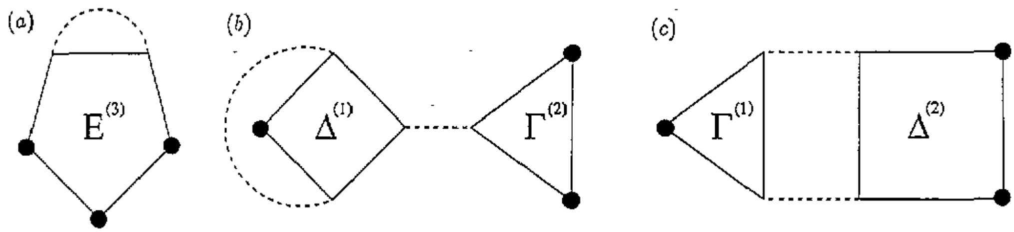

Figure 3. The new Gaussian corrections to the third-order vertex: (a) $E^{(3)}\mathcal{D}$, (b) $\mathcal{D}\Delta^{(1)}\mathcal{D}\Gamma^{(2)}$ and (c) $\Gamma^{(1)}\mathcal{D}\mathcal{D}\Delta^{(2)}$.

Although we have not attempted to explicitly calculate the contribution of the Gaussian correction terms to the Hall coefficient, we can comment on the form of the result, at least for the first two diagrams. Figure 3(a) depicts bare three-field slave vertices dressed with a single internal gauge-field propagator, which are the lowest-order perturbation correction to these vertices from the interaction of the slaves with this field. As we shall explain in section 4, terms such as these have already been implicitly assumed in the calculation of the vertices of the saddle-point action when we included transport lifetimes [20]. The term in $\Delta^{(1)}\mathcal{D}$ (figure 3(b)) will lead to two corrections, as may be seen from its contribution to the Hall current (δJ^H) the sum of which may be expressed as

$$\delta J_i^H = -6\frac{e^3}{c^2}\mathcal{D}_{ab}\mathcal{D}_{ef}\left[\left\{\Delta^{(1)}_{baei}\Gamma^{(2)}_{fjk} + \Delta^{(1)}_{baej}\Gamma^{(2)}_{fki}\right\} A_j^{(1)} A_k^{(2)} + \Delta^{(1)}_{baej}\Gamma^{(2)}_{fki} A_j^{(2)} A_k^{(1)}\right] \tag{22}$$

where $A^{(1)}$ and $A^{(2)}$ are as defined in section 2 and the current is evaluated in the limit $k/\omega \to 0$, where $\omega, k \to 0$ also. Therefore, we obtain

$$\delta R_H = -\frac{3}{e}\frac{1}{\chi_F + \chi_B}\left[2M\left\{\frac{\chi_F}{\sigma_F}R_H^B + \frac{\chi_B}{\sigma_B}R_H^F\right\} + \Lambda'\left\{R_H^B + R_H^F\right\}\right] \tag{23}$$

where Λ' and M are the k^2 and ω terms of $\Delta^{(1)}\mathcal{D}$ in the limits $\omega/k \to 0$ and $k/\omega \to 0$, respectively. Note that the second contribution is denoted by Λ' as it may be considered an amendment to the Λ term representing the Gaussian fluctuation correction to R_H found in [21]. The third diagram, figure 3(c), is more complicated as we must integrate over an internal momentum and therefore require a more detailed knowledge of the Γ's and Δ's. However, as shall be shown in the next section, this term, if it contributes at all to the hall conductivity, is $O(\delta^2)$, where δ is the doping (hole) concentration. Therefore, this term is negligible in the low doping limit and so we shall not pursue it any further.

4. Discussion of the Gaussian corrections

Thus far we have derived expressions, in terms of the various vertices of the slave fields, for the classical action of the external gauge field and the first quantum corrections to it through to third order in the field, where the latter terms arise from the Gaussian fluctuations of the internal field about its classical value. The vertices appearing, as derived from (1) by integrating out the slave fields, are strictly the bare vertices. However, as we have already noted, some of the terms in the Gaussian correction, e.g. the first terms in equations (20) and (21) are readily interpreted as the bare slave vertices dressed with a single gauge propagator. Since in principle it is possible to continue adding higher-order corrections, giving the complete series expansion for $S_{\rm eff}[A]$, we may postulate that a subset of the diagrams so obtained may be summed giving a contribution to $S_{\rm eff}$ with the same form as $S^c_{\rm eff}$, but expressed in terms of the fully dressed polarizations and propagator. Indeed, we implicitly assumed that this substitution of the bare vertices for the dressed ones is possible when discussing the Hall effect [20], while Lee and Nagaosa stated this explicitly [13]. A notable example is the conductivity where, from the Ioffe–Larkin action, it is found that $\sigma_{\rm eff} = \sigma_B\sigma_F/(\sigma_B+\sigma_F)$. Taking the slave conductivities to be Drude-like, i.e. $\sigma_s = n_s\tau_s/m_s$, then for $\sigma_B \ll \sigma_F$, $\sigma_{\rm eff} \simeq \sigma^B \propto T^{-1}$ [15, 26], in agreement with experiment, as the holon's transport time $\tau_B \sim T^{-1}$, which is a result of the holon being taken to be dressed by the gauge field, i.e. the dressed, not bare, two-point vertices are used.

However, it is clear that the above procedure does not account for all the graphs obtained from the Gaussian fluctuations and also, presumably, for the higher-order contributions. To be definite, let us consider the second term of Π^g in Eqn. (20), the presence of which was noted in [21]. This term cannot be written in the Ioffe–Larkin like form $\sum_s[\mathcal{D}\Pi^{\bar{s}}]^2\pi^s$, but instead has the form $\sum_{s_1,s_2}\zeta_{s_1}\zeta_{s_2}[\mathcal{D}\Pi^{\bar{s}_1}][\mathcal{D}\Pi^{\bar{s}_2}]\pi^{s_1s_2}$ $(s_i = B, F)$, where $\pi^{s_1s_2}$ represents a factor that contains a product of s_1- and s_2-slave responses. Such an expression therefore includes products of holon and spinon responses, which we shall call 'cross' terms. Consequently, this term represents a correction to the Ioffe–Larkin result, being qualitatively different to it. Thus, it would appear that on including higher-order corrections, the two-point vertex is given by an expression of the form $\Pi_{\rm eff} = \Pi^{IL}_{\rm eff} + \Pi^{\times}_{\rm eff}$, where $\Pi^{IL}_{\rm eff}$ is the Ioffe–Larkin part, as discussed above, and $\Pi^{\times}_{\rm eff}$ is the contribution from expressions leading to cross terms. Of course, such contributions are possible from (1), but none occur at the classical level.

As explained in [21], the cross term in Π^g does not effect the conductivity, but it does contribute to the diamagnetic susceptibility, an approximate calculation for which is given in the appendix as we obtain a slightly different result to that previously published. This correction is found to be $O(\delta^2)$, which is easy to understand heuristically if we note that the bosonic and fermionic vertices are typically $O(\delta)$ and $O(1-\delta)$, respectively, as functions of the doping. Since the cross terms contain at least two factors that are the product of a bosonic and a fermionic part, the overall expression will be $O(\delta^2)$. This should be contrasted with the Ioffe–Larkin term which may be similarly argued to be $O(\delta)$ and is therefore dominant in the low-doping limit [21].

However, we should compare the above with the current constraint approach [13, 31]. This gives a $\sigma_{\rm eff}$ and $\chi_{\rm eff}$ that are both in the Ioffe–Larkin form, i.e. no cross terms. Thus, while, at least to the Gaussian level, the two approaches agree as to the form of the conductivity, we obtain different results for the diamagnetic susceptibility.

For the three- and higher-point vertices we may likewise postulate a similar decomposition into an 'Ioffe–Larkin' and cross term contribution. It follows that, if the above doping arguments hold, in the low-doping limit the former term dominates and this appears to be the case, at least for the three-point vertex [21]. Consequently, for the Hall effect, we would regain the results obtained previously [20].

5. Conclusions

In this paper we have examined the effective action ($S_{\rm eff}$) of an electromagnetic field interacting with the model given by (1) by deriving first the classical (saddle-point) action, which we employed in a previous paper [20] and secondly including the Gaussian fluctuations about the classical field. We noted that for a starting action correct to nth order in the gauge fields we would obtain the classical action correct to nth order too, but the Gaussian correction only to $(n-2)$th order. We then examined the magnetoconductivity using the classical action, obtaining the same result as that obtained from the current constraint argument. We derived the Gaussian correction explicitly through to third order, noting the presence of 'cross' terms in the slave fields and proceeded to speculate about the form of the higher-order corrections. We noted that such terms lead to results that appear to disagree with the current constraint approach. Finally, we approximately calculated the second-order correction to the diamagnetic susceptibility from the cross term, obtaining a different result to that previously published [21].

6. Acknowledgments

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Appendix A. Approximate calculation of the cross-term contribution to $\chi_{\rm eff}$

If we denote by $\Pi^{\times}$ the cross term in Π^{g}, then we may express it as

$$\Pi^{\times}_{\lambda\mu}(p) = \sum_{s_1}\sum_{s_2}\zeta_{s_1}\zeta_{s_2}[\mathcal{D}\Pi^{\bar{s}_1}]_{\lambda\alpha}(p)[\mathcal{D}\Pi^{\bar{s}_2}]_{\mu\beta}(p)\tilde{\Pi}^{s_1s_2}_{\alpha\beta}(p) \tag{A1}$$

with

$$\tilde{\Pi}^{s_1s_2}_{\alpha\beta}(p) = -2\int\frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}T\sum_{b_m}\left[\bar{\Gamma}^{s_1}_{\sigma\nu\alpha}(q;-p-q)\bar{\Gamma}^{s_2}_{\tau\rho\beta}(p+q;-q)\mathcal{D}_{\rho\sigma}(q)\mathcal{D}_{\nu\tau}(p+q)\right] \tag{A2}$$

where $p=(\boldsymbol{p},b_n)$, $q=(\boldsymbol{q},b_m)$, $b_l=2\pi lT$ and

$$\Gamma^{s}_{\alpha\beta\gamma}(k_1;k_2;k_3) = [\delta_{k_1+k_2+k_3,0}(2\pi)^2/T]\bar{\Gamma}^{s}_{\alpha\beta\gamma}(k_1;k_2)$$

T being the temperature. Converting the Matsubara sum to an integral over the real frequency ω' and analytically continuing the free Matsubara frequency to ω, produces two terms in the integrand of the form $\bar{\Gamma}^{s}_{\alpha\beta\gamma}(\boldsymbol{p}+\boldsymbol{q},\frac{1}{\mathrm{i}}(\omega+\omega'+\mathrm{i}0^+);-\boldsymbol{q},-\frac{1}{\mathrm{i}}(\omega'+\mathrm{i}0^+))$. To calculate the contribution to $\chi_{\rm eff}$ we take the limits $\omega,k\to 0^+$, such that $\omega/k\to 0^+$. Therefore, we evaluate the Γs at $\omega=0$ and for convenience at $\boldsymbol{q}=\boldsymbol{0}$ too, as we can then express them in terms of the Hall conductivities (σ^{s}_{H}) using $\bar{\Gamma}^{s}_{\alpha\beta\gamma}(\boldsymbol{p},\frac{1}{\mathrm{i}}(\omega'+\mathrm{i}0^+);-\boldsymbol{p},0)=[\delta_{\alpha\beta}p_\gamma-\delta_{\beta\gamma}p_\alpha]\omega'\sigma^{s}_{H}/2$ [20]. It follows that $\tilde{\Pi}^{s_1s_2}$ is purely transverse, being

$$\tilde{\Pi}^{s_1s_2}_{T}(\boldsymbol{p},0^+) \simeq p^2\sigma^{s_1}_{H}\sigma^{s_2}_{H}\left\{-\frac{P}{2\pi^2}\int_{-\infty}^{\infty}\mathrm{d}\omega'\frac{\omega'^2}{e^{\omega'/T}-1}\int_0^{\infty}\mathrm{d}q\,q\,\mathrm{Im}\left[D^{R}_{L}(q,\omega')D^{R}_{T}(q,\omega')\right]\right\} \tag{A3}$$

with P indicating a principle value frequency integral while D_L^R and D_T^R are the longitudinal and transverse components of the retarded gauge-field propagator [32]. Denoting the term in braces by X, the contribution to $\chi_{\rm eff}$ of $\Pi^\times$ ($\chi_{\rm eff}^\times$) is

$$\chi_{\rm eff}^\times = X\left(\frac{\chi^F\sigma_H^B - \chi^B\sigma_H^F}{\chi}\right)^2 \tag{A4}$$

$\chi = \chi^B + \chi^F$ and so in the low doping limit $\chi_{\rm eff}^\times = O(\delta^2)$. This has a different form to that in [21] since with the present result, as $\delta \to 0$, the holons and spinons appear in a similar manner, while in the cited reference, only the term in the spinon Hall conductivity remains. This is because, in the latter calculation, the zero-doping limit was taken for what we have called $\tilde{\Pi}$ before the screening factors were included, giving an incorrect result.

X can be approximately calculated, but only at the expense of introducing an ultraviolet cut-off. Since, as we have noted, $\chi_{\rm eff}^\times = O(\delta^2)$ and therefore negligible in the low-doping limit, we shall not pursue this calculation any further here.

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[18] In [20] the holons interacted with the field $a + eA/c$ and the physical charge current was therefore given by $\delta S_{\rm eff}/\delta A$. This different sign is a matter of convention, but as the holons are positively charged, in the present paper we will couple them to the potential $a - eA/c$ so the current is now equal to $-c\delta S_{\rm eff}/\delta A$. Thus, because of these two sign changes our previous result for the Hall current is unchanged, despite this different sign convention.

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[24] The quadratic term in a^c does not contribute to the second- or third-order terms in $S_{\rm eff}^c$, which explains why this action can be obtained by merely shifting the internal field [21] from a to $a - \Pi^B \mathcal{D} A = a + \gamma^{(1)}A$, where $\gamma^{(1)}A$ is the classical field to first order in the external field.

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[30] In calculating $S^g_{\rm eff}$ some further diagrams are obtained, including one linear in A, all of which contain the factor $\mathcal{D}_{ab}\Gamma^{(1)}_{abi}$. As noted in [21], under static gauge transformations $\Gamma^s_{\alpha\beta\gamma}(k; -k; 0) = 0$ and therefore $\mathcal{D}_{ab}\Gamma^{(1)}_{abi} = 0$, too.

[31] Ioffe L B and Kalmeyer V 1991 *Phys. Rev.* B **44** 750

[32] Note that while only transverse components of D appear in [21], it has been shown that this result follows from an expression in which both transverse and longitudinal components appear; Schofield A J 1992 *PhD Thesis*, University of Cambridge.